An analytical solution for the rotational component of the Foundation Input Motion induced by a pile group

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16 **figures**

Keywords

Kinematic interaction, piled foundations, rotational component

List of Symbols

a_0, a_λ = dimensionless frequency parameters

a_b = acceleration at the base of the SDOF

a_{ff} = free-field acceleration

a_G = pile group acceleration

a_r = acceleration at bedrock level

a_{SK} = absolute structural acceleration considering kinematic interaction

a_{Sff} = absolute structural acceleration neglecting kinematic interaction (i.e. a_b=a_{ff} and θ_b=0)

B = distance between the two extreme piles in a row

c_S = viscous damping coefficient of a SDOF system

d = pile diameter

E_p = pile Young's modulus

E_s = soil Young's modulus

$f(n)$ = dimensionless function of number of piles

h = height of a SDOF system

H = horizontal force at the pile head

$H(\omega)$, $F(\omega)$ = transfer functions of the SDOF system

I_p = cross-sectional moment of inertia of pile

I_u, I_{uX}, I_{uY} = horizontal kinematic interaction factors of the foundation

I_{uR} = horizontal kinematic interaction factor of the fixed-head pile

I_{uS} = horizontal kinematic interaction factor of the free-head pile

$I_{\theta S}$, $I_{\theta S\lambda}$ = rotational kinematic interaction factors of the single pile

L = pile length

I_θ , $I_{\theta G}$, $I_{\theta X}$, $I_{\theta Y}$ = rotational kinematic interaction factors of the foundation

J_a = ratio of the mass acceleration generated by kinematic interaction over that of free-field

k = Winkler modulus

k_S = stiffness of a SDOF system

K_V, K_{HH}, K_{HM}, K_{MM} = axial, swaying, cross-swaying and rotational stiffness of single pile

M, M_Y = restraint moments at the pile head

m, n, p = number of piles

m_S = mass of a SDOF system

N, N_i = axial loads on piles

s = pile spacing

T_{st} = oscillation period of the fixed-base building

u, u_X, u_Y = horizontal displacements of the foundation

u_b = horizontal displacement at the base of the SDOF

u_{ff} = free-field horizontal displacement

u_M , u_{MX}= single pile horizontal displacements due to moment restraint

u_R = fixed-head single pile horizontal displacement

u_S = free-head single pile horizontal displacement

u_Z, u_{Zi} = vertical displacements of piles

V_s = shear wave propagation velocity in the soil

x_G, y_G = center of pile group vertical stiffness

x_i, y_i = coordinates of i-th pile

β_s = soil damping ratio

χ = dimensionless factor for a row of piles

λ = Winkler wavenumber

θ, θ_G, θ_Y = pile cap rotations

$\theta_b, \ddot{\theta}_b$ = rotation and rotational acceleration at the base of the SDOF

θ_M , θ_{MY} = single pile rotation due to restraint moment

θ_s = single pile rotation

ω = excitation frequency

ξ = dimensionless factor for a pair of piles

ξ_X, ξ_Y = dimensionless factors for a group of piles

Abstract

This work investigates the effect of the rotational component of motion induced by the kinematic interaction between a pile group and the surrounding soil on the seismic behaviour of a structure. To this end, a simple analytical model is developed by deriving the pile group behaviour from the seismic response of a single pile, taking into account equilibrium and compatibility of displacements at the level of the piles' heads. Closed-form solutions in the frequency domain are provided for both the translational and the rotational motion of a group of unevenly distributed identical piles, rigidly connected at the top and displaced by the surrounding soil, which is subjected to purely translational oscillations. The proposed solutions, applicable to any subsoil conditions, highlight that pile group layout is the crucial parameter governing the magnitude of the foundation rotation. Further, new transfer functions from the soil surface in free field conditions to the top of a SDOF system are introduced, which take into account the translational and/or rotational kinematic effects. An application of the above concepts to a case study is presented, highlighting that the rotational component of input motion may be important for tall structures on small pile groups.

1. INTRODUCTION

The seismic analysis of a structure can be conveniently carried out through the well-known substructure method (Kausel *et al*. 1978; Wolf & von Arx, 1978; Gazetas, 1984; Makris *et al*. 1996; Mylonakis *et al*., 1997), consisting of three consecutive steps: (i) calculation of the seismic motion at foundation level, or the Foundation Input Motion (FIM), neglecting the mass of the superstructure; (ii) computation of the dynamic impedances ('springs' and 'dashpots') associated with swaying, vertical, rocking and cross swaying-rocking oscillation of the foundation; (iii) evaluation of the response of the superstructure supported on springs and dashpots determined in step (ii) and subjected to the FIM calculated in step (i).

The substructure method is most commonly adopted in practice by assuming that the foundation motion is equal to the free-field seismic input. For a piled foundation, this suggests that the change in the seismic input motion due to the presence of the piles is not accounted for, even if the horizontal displacements at the top of the piles may differ substantially from the free-field motion, especially for soft soils, where piles are frequently used to increase bearing capacity and/or reduce foundation settlement (Randolph, 1994; Kaztenbach *et al*. 1997; Russo *et al*. 2004; Russo & Viggiani, 1998; Viggiani, 2001). However, available evidence (Kawamura *et al.* 1977; Tajimi *et al*., 1977; Otha *et al*. 1980; Gazetas, 1984) demonstrates that piles may modify substantially the amplitude of the free-field ground acceleration, as high frequency components of the free-field motion are filtered out by pile-soil interaction. In addition to the above effects, soil-structure interaction induces a rotational component in the input motion, which does not exist in the corresponding free-field motion.

The kinematic response coefficient, intended as the ratio of the horizontal pile displacement over that of the free-field, was probably introduced by Blaney *et al*. (1976), who investigated the effect of a free-head pile on the motion at the free surface of an homogeneous soil deposit using the consistent boundary matrix method developed by Kausel *et al*. (1975). Since then, literature on pile-soil kinematic interaction effects concentrated primarily on the evaluation of the horizontal displacements and the rotation of a single pile, while a few works dealt with the rocking motion of pile groups. The early contribution on the problem of the rocking motion induced by kinematic interaction dates back to Wolf & von Arx (1978), who examined the kinematic response of groups of piles connected by a rigid mat using a continuum model with hysteretic and radiation damping. Results undertaken by the FEM in the frequency domain showed that the rocking motion at foundation level may be important, especially in the case of small groups of piles and high frequency content of the base excitation, while for large group of piles this component may be neglected. Gazetas (1984) applied the consistent boundary matrix method developed by Kausel *et al*. (1975) and later used by Blaney *et al*. (1976) to study the influence of a number of factors on the kinematic rotation of a single pile, including pile-soil stiffness ratio, soil inhomogeneity, soil damping ratio, and pile slenderness. Mamoon & Banerjee

(1990) implemented a hybrid boundary element method to study the problem of pile-soil kinematic interaction; their rotational kinematic interaction factors compare well with those obtained by Gazetas (1984). Fan *et al*. (1991) studied the kinematic behaviour of groups of vertical floating piles connected by a rigid massless cap. Pile-soil and pile-pile interaction were modelled rigorously, using the formulation by Kaynia & Kausel (1982). The results of their study indicate that the number of piles and their layout do not affect the horizontal component of the cap motion, *i.e*. that group effects are negligible for lateral displacements, while they affect significantly the rotation of the pile cap. This is always smaller than the rotation of the single pile, reduces with increasing spacing, and is affected only by the number of piles and their relative spacing parallel to the direction of the seismic excitation. Nikolaou *et al*. (2001) derived a closed form solution of the rotation of a long pile in a homogeneous soil by using the classical dynamic Winkler formulation. Mylonakis *et al*. (2001) applied the analytical solution by Nikolaou *et al*. (2001) to examine the seismic response of a single-span bridge supported by piers extended into the ground in the form of long-drilled shafts (single piles), concluding that the rocking motion caused by kinematic interaction may increase the seismic response of tall piers. Following Nikolaou *et al*. (2001), Anoyatis *et al*. (2013) provided analytical solutions for the rotation of a single pile embedded in a homogeneous soil layer using the classical Winkler formulation with different boundary conditions. Finally, Sextos *et al*. (2015) investigated the seismic performance of bridge piers supported by groups of 2^2 piles embedded in a homogeneous soil layer with shear wave velocities of V_s = 360 m/s or 180 m/s. The kinematic response of the foundation was analysed by placing the individual piles on uniformly distributed frequency-dependent springs and dashpots along the length. Results of this study showed that, in case of soft soil, the rotational component induced by piles may lead to an increase of the deck displacement ranging between 1.2 and 1.7.

Besides these results, and despite the fact that the Eurocodes (EN 1998-5, 2004) prescribe that potential negative consequences of the rocking motion of the foundation should be taken into account, to the knowledge of the Authors, no analytical solutions have been developed to quantify the kinematic rotational motion of a pile group and evaluate its consequences on the structural behaviour. As a contribution to this topic, this work aims at: (i) offering insight into the mechanism of kinematically induced rotation of a group of piles connected by a cap; (ii) developing a simple closed-form solution for the rotational component of the FIM to be applied at the base of a structure founded on piles; and (iii) providing guidelines to evaluate the importance of the rotational component of the FIM.

2. PROBLEM STATEMENT

Figure 1 defines the problem under examination. The acceleration applied at the bedrock, a_r, is transferred to the ground surface in free-field conditions as a_{ff}; due to the kinematic interaction between the pile group and the soil, the foundation seismic motion has a horizontal component, a_G, and a rotational component, $\ddot{\theta}_G$, generated by the rigid connection between the piles (Figure 1a). The acceleration time histories, a_G and $\ddot{\theta}_G$, are considered as the seismic input in the inertial interaction analysis of the structure, (Figure 1b) and will be investigated in the following.

3. KINEMATIC RESPONSE OF A SINGLE PILE

The change of motion at the head of a pile free-to-rotate is usually specified through the following transfer functions (Kaynia & Kausel, 1982):

$$I_{uS} = \frac{u_s(0,\omega)}{u_{ff}(0,\omega)} \tag{1}$$

$$I_{\theta S} = \frac{\theta_s(0,\omega)d}{u_{ff}(0,\omega)} \tag{2}$$

where u_s and θ_s are the displacement and the rotation of the pile head, u_{ff} is the free-field horizontal displacement and ω is the angular excitation frequency. More recently, Anoyatis *et al*. (2013) expressed the rotational kinematic interaction factor of a single pile embedded in a homogeneous soil as:

$$I_{\theta S\lambda} = \frac{\theta_s(0,\omega)}{\lambda u_{ff}(0,\omega)} \tag{3}$$

$$\lambda = \left(\frac{k}{4E_p I_p}\right)^{1/4} \tag{4}$$

where λ is the Winkler wave number, E_p and I_p are the Young's modulus and the cross-sectional moment of inertia of the pile, respectively, k [F/L^2] is the Winkler modulus of sub-grade reaction, related to the soil's Young's modulus E_s through a proportionality coefficient δ (Kavvadas & Gazetas, 1993; Mylonakis, 2001). Equation (4) is real-valued and, therefore, rigorous only under static conditions; however, the use of a static wave number also for dynamic conditions offers two advantages over the corresponding complex-valued parameter: (i) Winkler solutions are in better agreement with rigorous numerical results; (ii) the introduction of a real-valued wave number allows to define proper dimensionless quantities thus providing an insight into the physics of the interaction phenomenon.

Anoyatis *et al.* (2013) provided complex-valued closed form expressions of I_{uS} and $I_{\theta S\lambda}$ for any pile length L. The advantage of the new normalization is that the kinematic interaction factors are functions of solely two parameters, namely the 'mechanical slenderness' λL, merging pile geometrical slenderness L/d and pile-soil stiffness ratio E_p/E_s, and the dimensionless frequency $a_\lambda = \omega/(\lambda V_s)$.

Interaction factors I_{uS} and $I_{\theta S\lambda}$, are complex-valued for non-zero values of damping ratio. However, despite both soil and pile response are affected by soil damping, pile-soil interaction is almost insensitive to it, as shown by Di Laora & de Sanctis (2013), and therefore can be conveniently expressed through real-valued functions without significant loss of accuracy. Figure 2 summarises the effects of a_λ on the kinematic interaction factors I_{uS} and $I_{\theta S}$ for values of λL ranging between 2 and infinity. From the plots in Figure 2 it is evident that, provided that the 'mechanical slenderness' λL is larger than 3, the pile is sufficiently long and a further increase in length does not affect I_{uS} and $I_{\theta S}$.

For an infinitely long free-head pile kinematic interaction factors in translation and rotation depend solely on the frequency parameter a_λ and take the simple form (Rovithis *et al*. 2015):

$$I_{uS} = \left(1 + \frac{1}{2} a_\lambda^2\right) I_{uR} = \frac{1 + \frac{1}{2} a_\lambda^2}{1 + \frac{1}{4} a_\lambda^2} \tag{5}$$

$$I_{\theta S \lambda} = a_\lambda^2 I_{uR} = \frac{a_\lambda^2}{1 + \frac{1}{4} a_\lambda^2} \tag{6}$$

where I_{uR} is the kinematic interaction factor for the fixed-head pile.

For piles embedded in a multi-layered soil, closed form solutions for kinematic interaction factors get very complicated, and the use of a general BDWF approach or, alternatively, a numerical method is preferable. In this case, normalisation of the rotational interaction factor by the pile diameter (Eq. 2) is preferable due to the inherent difficulty in the definition and computation of the parameter λ.

In the following sections, starting from the behaviour of free-head single piles embedded in either homogenous or layered soils and making use of equilibrium and compatibility of displacements at the piles' heads, the kinematic interaction factors for a group of piles connected by a rigid cap are derived.

4. KINEMATIC RESPONSE OF PILE GROUPS

Pair of piles

The rotation of a cap connecting rigidly the top of a pair of piles at a spacing s may be decomposed as the sum of the kinematic rotation of a free-head pile and the rotation due to the internal forces arising from the conditions of compatibility of displacements and rotations at top of the piles, as illustrated in Figure 3. Owing to the absence of relevant group effects for kinematic loading (Kaynia & Kausel, 1982; Fan *et al.*, 1991; Maiorano *et al.*, 2009), substructure (a) can be easily solved considering an isolated free-head pile. On the other hand, substructure (b) is excited by the internal forces applied at the top of the piles, and therefore, in principle, group effects are not negligible. However, group effects associated with the rocking oscillation are of minor concern compared to vertical loading and, therefore, the dynamic interaction between the piles is neglected in this work for sake of simplicity. The validity of this approximation is checked against benchmarks obtained through 3D boundary element analyses, as it will be shown in Section 5.

According to the decomposition shown in Figure 3, the rotation, θ, and the horizontal displacement, u, at the top of the two pile group can be written as:

$$\theta = \theta_s + \theta_M \tag{7}$$

$$u = u_s + u_M \tag{8}$$

where θ_s and u_s are the rotation and the horizontal displacement of the top of the single pile under kinematic loading, and θ_M and u_M are those due to the moment at the top of the piles. It can be shown that:

$$\theta_M = -\frac{\xi}{1+\xi}\theta_s \tag{9}$$

$$u_M = \frac{\xi}{1+\xi}\frac{K_{HM}}{K_{HH}}\theta_s \tag{10}$$

where ξ is a dimensionless factor defined as:

$$\xi = \frac{1}{4}\frac{K_V s^2}{-\frac{K_{HM}^2}{K_{HH}} + K_{MM}} \tag{11}$$

and K_V, K_{HH}, K_{HM} and K_{MM} are the vertical, swaying, cross swaying-rotational and rotational stiffness of the single pile, respectively (see APPENDIX A).

Thus, the final expressions for the rotation and the horizontal displacement of the pile cap become:

$$\theta = \frac{\theta_s}{1+\xi} \tag{12}$$

$$u = u_s + \frac{\xi}{1+\xi}\frac{K_{HM}}{K_{HH}}\theta_s \tag{13}$$

The case of infinite axial stiffness leads to:

$$\theta_M = -\lim_{K_z\to\infty}\frac{\xi}{1+\xi}\theta_s = -\theta_s \tag{14}$$

$$u_R = u_s + \frac{K_{HM}}{K_{HH}}\theta_s \lim_{K_z\to\infty}\frac{\xi}{1+\xi} = u_s + \frac{K_{HM}}{K_{HH}}\theta_s \tag{15}$$

corresponding to the situation where the rotation of the cap is fully constrained ($\theta = 0$).

Note that, as the two terms on the right-hand side of Eq. (13) have opposite sign (see Figure 3), u is always smaller than u_s in absolute terms; upon comparing Eqs. 13 and 15, as $\xi/(1+\xi) < 1$, u is always larger than u_R in absolute terms. For piles fully restrained against rotation, foundation motion is not affected by pile spacing and is identical to that of the single pile (Eq. A1 in Appendix A). Although this is a direct consequence of the simplifying assumptions of the proposed method, the same conclusion was reached when pile-to-pile interaction was explicitly modelled (Fan *et al*. 1991).

By combining Eqs. (13) and (15) the following relationship is found:

$$I_u = \frac{\xi}{1+\xi}I_{uR} + \frac{1}{1+\xi}I_{uS} \tag{16}$$

Thus, the translational kinematic factor of a pair of piles, I_u, is a weighted average between the kinematic factors of the fixed-head and the free-head single pile.

Note that the above expressions are valid in any subsoil condition. For the particular case of a long pile embedded in homogenous soil, the swaying, cross swaying-rotational and rotational stiffness components can be expressed as follows (Hetenyi, 1946; Pender, 1993; Mylonakis, 1995):

$$K_{\mathrm{HH}} = 4E_{\mathrm{P}}I_{\mathrm{P}}\lambda^{3} \tag{17}$$

$$K_{\mathrm{HM}} = 2E_{\mathrm{P}}I_{\mathrm{P}}\lambda^{2} \tag{18}$$

$$K_{\mathrm{MM}} = 2E_{\mathrm{P}}I_{\mathrm{P}}\lambda \tag{19}$$

Substituting these expressions into Eq. (11) yields:

$$\xi = \frac{1}{4}\frac{K_{\mathrm{V}}s^{2}}{E_{\mathrm{p}}I_{\mathrm{p}}\lambda} = \frac{1}{2}\frac{K_{\mathrm{V}}s^{2}}{K_{\mathrm{MM}}} \tag{20}$$

In summary, the amplitude of the rotational component of the FIM is affected by the axial and rotational stiffness of the single pile and by pile spacing. Parameter ξ in Eq. (20) quantifies the importance of the rotational component, in the sense that the larger the ξ, the smaller the amplitude of the rotational component of the FIM.

Figure 4 shows the kinematic interaction factor $I_{\mathrm{u}} = u/u_{\mathrm{ff}}$ against the classical dimensionless frequency parameter $a_0 = \omega d/V_{\mathrm{s}}$ for different values of pile spacing, s/d. The horizontal displacement was calculated using Eq. (15) or (13), depending on whether the cap is fully restrained against rotation. Pile axial stiffness K_{V} was evaluated by means of the classical Randolph and Wroth (1978) formula considering only shaft contribution.

The kinematic interaction factor I_{u} can be reasonably replaced by $I_{\mathrm{uR}} = u_{\mathrm{R}}/u_{\mathrm{ff}}$ for $s/d \geq 5$, which is consistent with the results by Fan *et al.* (1991). Coefficient ξ has a rather small effect on the horizontal component of the FIM, whereas it affects remarkably its rotational component. As already outlined, if the axial stiffness tends to infinity, the rotational component is zero; from a practical standpoint this is the case of relatively short, end-bearing piles. In all other cases the rotational component depends on the combination of the single pile stiffness components (K_{V}, K_{HH}, K_{HM}, K_{MM}).

The dependency of the stiffness components on frequency is usually negligible, and hence the ratio $\theta/\theta_{\mathrm{s}}$ does not depend on the dimensionless term a_0, see Eq. (12). Figure 5 shows the dependency of the ratio θ/θ_s on relative pile spacing, *s/d*, for different values of pile slenderness, *L/d,* and pile-soil stiffness ratio, E_p/E_s; in this case θ/θ_s was computed using the static values of single pile stiffness. For any given pile spacing, the amplitude of the rotational component decreases with increasing pile slenderness and pile-soil stiffness ratio E_P/E_S.

Groups of equally spaced identical piles

Eqs. (12) and (13) can be easily extended to a group of $m\times n$ equally spaced piles, where n is the number of piles along the direction of the seismic action. Fan et *al*. (1991) demonstrated that the number of piles perpendicular to the direction of the seismic action does not affect kinematic response, and therefore the problem of a group of equally spaced piles can be reduced to a row of piles at spacing s. This problem is shown schematically in Figure 6.

Moment equilibrium can be written as:

$$nmM_Y = \sum_{i=1}^{p} N_i x_i = m\sum_{i=1}^{n} N_i x_i \tag{21}$$

where $p = nm$, or:

$$nM_Y = \sum_{i=1}^{n} N_i x_i \tag{22}$$

The components of the foundation's motion are readily derived as (APPENDIX B):

$$\theta_Y = \frac{\theta_s}{1+\chi} \tag{23a}$$

$$u_X = u_s + \frac{\chi}{1+\chi}\frac{K_{HM}}{K_{HH}}\theta_s \tag{23b}$$

where:

$$\chi = \xi\frac{n^2-1}{3} \tag{24}$$

For a long pile embedded in a homogeneous soil, ξ can be obtained from Eq. (20), and hence Eq. (24)

can be written in the form:

$$\chi = \frac{1}{2}\frac{K_{\mathrm{V}} s^2}{K_{\mathrm{MM}}}\frac{n^2 - 1}{3} \tag{25}$$

Thus, the amplitude of the rotational component of the FIM decreases with increasing single pile axial stiffness, pile spacing and number of piles. From the above expressions, it is straightforward to verify that for $n = 1$, factor χ is zero and therefore the interaction factors of the group reduce to the corresponding kinematic interaction factors for a free-head single pile, and, finally that, for $n = 2$, $\chi = \xi$, and the expressions for the pair of piles are obtained.

The distance between the two external piles, B, can be expressed as:

$$B = s(n-1) \tag{26}$$

Substituting into Eq. (25):

$$\chi = \frac{1}{6}\frac{K_{\mathrm{V}} B^2}{K_{\mathrm{MM}}}\frac{n+1}{n-1} \tag{27}$$

Equation (27) reveals an important aspect of the response: the amplitude of the rotational component is strongly affected by the extension of the pile group along the direction of shaking, B; by contrast the number of piles in the group has only a small effect.

The kinematic interaction factors for a pile group can be calculated as a function of the corresponding interaction factors for a single pile:

$$I_{\mathrm{uX}} = \frac{u_{\mathrm{X}}}{u_{\mathrm{ff}}} = I_{\mathrm{uS}} + \frac{\chi}{1+\chi}\frac{K_{\mathrm{HM}}}{dK_{\mathrm{HH}}} I_{\theta\mathrm{S}} \tag{28a}$$

$$I_{\theta\mathrm{Y}} = \frac{\theta_{\mathrm{Y}} d}{u_{\mathrm{ff}}} = \frac{I_{\theta\mathrm{S}}}{1+\chi} \tag{28b}$$

Figure 7 illustrates the relationship between the maximum values of I_{uX} and $I_{\theta\mathrm{Y}}$ across the whole range of frequencies, with varying parameter χ, obtained by means of a numerical procedure. As expected, $I_{\theta\mathrm{Y}}$ is strongly affected by χ which, in turn, has a negligible influence on kinematic interaction factor I_{uX}.

Group of unevenly distributed identical piles.

The procedure outlined above can be extended to the case of a group of unevenly distributed identical piles. Figure 8 illustrates the problem under examination and introduces the reference system; (x_{G},y_{G}) are the coordinates of the centre of the vertical stiffness, *i.e.*, the point where application of a vertical load would not produce rotation.

Moment equilibrium around the *Y*-axis yields:

$$pM_{\mathrm{Y}} = \sum_{i=1}^{p} N_i x_i \tag{29}$$

where M_{Y} is the bending moment around the *Y*-axis at the pile head, N_i the axial load applied on the *i*-th pile, and p the total number of piles.

Eq. (29) can be rewritten as:

$$pM_{\mathrm{Y}} = K_{\mathrm{V}} \sum_{i=1}^{p} u_{\mathrm{Z}i} x_i = -K_{\mathrm{V}} \theta_{\mathrm{Y}} \sum_{i=1}^{p} \left(x_i^2 - x_i x_{\mathrm{G}} \right) \tag{30}$$

where θ_{Y} is the rotation around the *Y*-axis.

The overall rotation around the *Y*-axis can be expressed according to Eq. (7):

$$\theta_{\mathrm{Y}} = \theta_{\mathrm{s}} + \theta_{\mathrm{MY}} = \theta_{\mathrm{s}} + \frac{M_{\mathrm{Y}}}{-\frac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \tag{31}$$

where θ_{MY} is the counter-rotation of the single pile head due to moment M_{Y}. The kinematic interaction coefficients along *X* and *Y* can be derived as (APPENDIX C):

$$I_{\mathrm{uX}} = \frac{u_{\mathrm{X}}}{u_{\mathrm{ff}}} = I_{\mathrm{uS}} + \frac{\xi_{\mathrm{Y}}}{1+\xi_{\mathrm{Y}}} \frac{K_{\mathrm{HM}}}{dK_{\mathrm{HH}}} I_{\theta\mathrm{S}} \tag{32a}$$

$$I_{\theta\mathrm{Y}} = \frac{1}{1+\xi_{\mathrm{Y}}} I_{\theta\mathrm{S}} \tag{32b}$$

$$I_{\mathrm{uY}} = \frac{u_{\mathrm{Y}}}{u_{\mathrm{ff}}} = I_{\mathrm{uS}} + \frac{\xi_{\mathrm{X}}}{1+\xi_{\mathrm{X}}} \frac{K_{\mathrm{HM}}}{dK_{\mathrm{HH}}} I_{\theta\mathrm{S}} \tag{33a}$$

$$I_{\theta\mathrm{X}} = \frac{1}{1+\xi_{\mathrm{X}}} I_{\theta\mathrm{S}} \tag{33b}$$

where:

$$\xi_{\mathrm{Y}} = \frac{K_{\mathrm{V}}}{-\dfrac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \sum_{i=1}^{p} \frac{x_i^2 - x_{\mathrm{G}} x_i}{p} \tag{34a}$$

$$\xi_{\mathrm{X}} = \frac{K_{\mathrm{V}}}{-\dfrac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \sum_{i=1}^{p} \frac{y_i^2 - y_{\mathrm{G}} y_i}{p} \tag{34b}$$

$$x_{\mathrm{G}} = \sum_{i=1}^{p} \frac{x_i}{p} \tag{35a}$$

$$y_{\mathrm{G}} = \sum_{i=1}^{p} \frac{y_i}{p} \tag{35b}$$

It is worth mentioning that the above methodology can be readily extended to the general case of piles with different properties. This however is beyond the scope of this work.

5. VALIDATION OF THE PROPOSED ANALYTICAL PROCEDURE

The closed form solutions obtained in this work - Eqs. (31) and (32) - are ready-to-use formulas for the evaluation of the two components of the seismic motion. The above solutions have been developed by assuming that the dynamic interaction between piles is negligible under internal forces generated by the cap. The dynamic pile-to-pile interaction is frequency dependent and is a consequence of waves emitted from the periphery of each pile, propagating until they reach the other piles. At zero frequency, both vertical and rocking dynamic group stiffness reduce to the respective static group stiffness. For static conditions, group effects are particularly relevant under vertical loading (Poulos, 1968; Butterfield & Banerjee, 1988; Viggiani, 2000), while they are less important under moment loading. The latter is also valid under dynamic loading since the stiffness component of the rocking impedance of a pile group is unaffected by excitation frequency (Dobry & Gazetas, 1988). The proposed analytical solution, thus, is deemed as sufficiently accurate for engineering purposes. The last statement is supported by the results plotted in Figure 9, where the proposed analytical solution is compared against benchmarks obtained using the dynamic BEM code by Kaynia & Kausel (1982). The dashed lines were obtained computing the axial stiffness and the rotational stiffness of the single pile, K_V and K_{MM}, with the Winkler approach, while the continuous lines were obtained computing the two stiffness components directly by the rigorous approach, with a slight improvement of the agreement with the reference solution. The difference with the BEM approach is only significant for pile spacing $s/d = 3$, where group effects are of major concern, and relatively high excitation frequencies. At $s/d = 5$ and/or for greater values of the pile spacing, the agreement with the rigorous numerical analysis is certainly satisfactory.

6. RESPONSE OF A SDOF INCLUDING THE ROTATIONAL COMPONENT OF THE BASE MOTION

Suppose that a Single Degree Of Freedom (SDOF) system, characterized by mass m_S, stiffness k_S and viscous damping coefficient c_S, is excited by a base acceleration, $(-\omega^2 u_b)$, and a rotational acceleration $(-\omega^2 \theta_b)$, both generated by pile-soil kinematic interaction. The absolute structural acceleration of the oscillating mass, a_{SK}, can be compared to that generated under free-field assumptions, a_{Sff}. It is easy to verify that the ratio of the two accelerations can be expressed as (APPENDIX D):

$$J_a = I_u - \frac{h}{d} I_\theta \tag{36}$$

where h is the height of the mass, while I_u and I_θ are defined as:

$$I_u = \frac{u_b}{u_{ff}} \tag{37a}$$

$$I_\theta = \frac{\theta_b d}{u_{ff}} \tag{37b}$$

In the special case of long piles in homogeneous soil J_a depends only on parameters a_λ, χ and λh , the latter increasing with structure height and decreasing with pile diameter, and may be expressed through the equation:

$$J_a = \frac{1 + \frac{1}{2} a_\lambda^2 + \frac{\left(\frac{\chi}{2} - \lambda h\right)}{1 + \chi}}{1 + \frac{1}{4} a_\lambda^2} \tag{38}$$

From a practical point of view, the classical response spectra under free-field conditions can be conveniently replaced by that obtained through Eq. (36), where the kinematic interaction factor I_u and I_θ can be evaluated from Eq. (32) or (33), depending on the direction of the earthquake. Noticeably, the ratio between the responses of the SDOF considering or not the interaction does not depend only on pile-soil kinematic interaction, but also on the height of the oscillator mass.

Figure 10 illustrates the relationship between the amplitude of J_a and the normalized frequency a_0 for the case of a row of 3 and 5 piles embedded in homogeneous soil, by varying pile spacing and dimensionless factor λh. The rotational component always implies an increase of the spectral acceleration and tends to compensate the reduction of the horizontal acceleration caused by pile-soil kinematic interaction. In the case of very few piles, small spacing and tall structures, the acceleration of the oscillator might be higher than in free-field conditions. However, the effects of the rotational component on the mass acceleration strongly diminishes by increasing number of piles and pile spacing.

7. APPLICATION TO A CASE STUDY

The proposed method is applied here to a case study of a pile-supported structure where the rotational component might be of importance. The case of an 11-storey building in Japan (Tajimi, 1977; Otha *et al*. 1980; Gazetas, 1984) is analysed (Figure 11). The building under examination is supported by a 2×14 cast-*in-situ* piles embedded in alluvial deposits consisting of alternating layers of sand and silt. Pile spacing along the weak direction is 8.35 m, while pile diameter is 1.4 m, or a ratio $s/d \cong 6$. The vibrations of the system due to seven earthquakes were monitored using 27 accelerometers, placed on the axis of the building, on an alignment 5 m away from the piles, and on an alignment 35 m away from the piles, representing free-field conditions. These seven earthquakes can be classified as small magnitude, near-field events ($M_L \leq 5$ and $R < 40$ km) and moderate and large-magnitude far-field events ($M_L \geq 5$ and $R > 65$ km). Acceleration recordings for the above events were published first by Ohta *et al*. (1980) and later on re-examined by Gazetas (1984). Figure 12 illustrates the distribution of the peak values of accelerations recorded on building and free-field alignments during the seven earthquakes. The ratio between the maximum pile-head acceleration, a_G, and the free field acceleration, a_{ff}, is about 0.6 for near-field events, and about 1.0 for far-field events. This depends on the fact that near-field events are usually very rich in high frequencies and pile-soil kinematic interaction filters out the high frequency components of the input signal. This work demonstrates the relevance of the filtering action exerted by the piles from an experimental point of view. However, the Author does not address the role of the rocking oscillation of the foundation on the overall seismic behaviour of the structure; by contrast, the effects of this component might be of importance, as this is the case of a relatively tall building supported by only two piles in the weak direction.

Frequency domain response

As per the analytical procedure suggested in the previous section, the kinematic response of the pile group is analysed in the frequency domain along the direction parallel to the alignment with two piles. To this aim, the response of the single pile is analysed first with the Beam on Dynamic Winkler Foundation (BDWF) formulation, and accounting for soil layering. The horizontal and rotational component of the pile group are then evaluated by using Eqs. (12) and (13), obtained for a pair of piles. Figure 13 shows the response of the pile group. The stiffness components of the single pile were computed: (a) considering their complex values, (b) considering only the real parts of the stiffness components (K_{real}), and (c) disregarding their imaginary parts and taking the real parts constant with frequency and equal to the corresponding static values (K_{static}). Noticeably, the imaginary parts of K_{HH}, K_{HM} and K_{MM} have very little effect on both I_u and I_θ. Also, the frequency dependency of the real parts does not affect the pile group response and therefore, from a practical point of view, the static terms of K_{HH}, K_{HM} and K_{MM} can be retained in Eqs. (12) and (13). In the same figure, the frequency domain response of the fixed-head single pile is also plotted for comparison. The axial stiffness was also taken as constant and equal to its static value, yet a BEM code accounting for soil layering through the Steinbrenner (1934) approximation was used. The horizontal displacement of the pair of piles is practically the same as that of the fixed-head single pile, as the amplitude of ratio $\xi/(1+\xi)$ is approximately equal to one, see Eq. (13).

Finally, Figure 14 shows the comparison between I_u and J_a. As per Figure 10, the rocking motion always leads to an increase of the spectral acceleration with respect to the case in which the filtering effect is accounted for with no cap rotation. For very low frequencies, the absolute value of J_a is above unity, and hence the spectral acceleration accounting for foundation rocking is expected to be larger than that associated with the free-field condition at least at structural periods corresponding to low frequencies.

Time domain response

Three natural earthquakes were selected for time domain analysis from the ESD (*European Strong Motion Database*, Ambraseys *et al*., 2002), namely Irpinia (Southern Italy, 1980), Kozani (Greece, 1995) and Nocera Umbra (Central Italy, 1997) records. The main characteristics of the above input signals are summarised in Table I, while Figure 15 shows their acceleration time histories and their

Fourier amplitude spectra. For the latter two signals, low- and high- frequency components are both significant.

The computed ratios of the maximum pile acceleration over that of the free-field are in good agreement with those obtained experimentally by Gazetas (1984), as shown in Figure 12. However, while the experimental ratios include the inertial effects due to the oscillation of the building, the calculated values are due solely to kinematic interaction. Thus, the satisfactory agreement in Figure 12 is supporting the idea offered by Ohta *et al*. (1980) and Gazetas (1984) that the reduction of maximum acceleration is due essentially to the filtering action exerted by the piles.

Figure 16 shows the results obtained in the time domain for the three recordings. The filtering action generated by the piles, intended as the reduction of the horizontal acceleration of the base motion, is evident for Nocera Umbra and Kozani, with significant high frequency components, while it is not very important for Sturno. The oscillation period of the fixed base building estimated by Gazetas is T_{st} = 1/1.83 = 0.55 s. The reduction of spectral acceleration at this structural period is negligible for all input signals when the response spectrum is evaluated with no cap rotation. On the other hand, the spectral acceleration of the oscillating mass increases by 7-10% of that corresponding to free-field conditions when the complete kinematic interaction including cap rotation is explicitly modelled. This is due to the occurrence of a resonance condition between the multi-storey building and the vibrational rocking motion of the foundation. Such result is fully compatible with plots in Figure 14. For large piled rafts the effect of the rotational component is expected to be negligible, because the magnitude of the rotation at foundation level decreases quadratically with B, as shown in Eq. (26).

8. CONCLUSIONS

This work investigated the rotational component of the Foundation Input Motion generated by the kinematic interaction between a group of piles connected by a rigid cap and the surrounding soil when subjected to earthquake shaking. In fact, despite the fact that codes prescribe that potential negative consequences of the rocking motion of the foundation should be accounted for, no simple methods are available to quantify the rotational component of the FIM and evaluate its consequences on the structural behaviour.

A novel closed-form analytical solution for both the horizontal and rotational component of a capped pile group embedded in any type of subsoil was derived. The starting point was the kinematic response of the single pile in the frequency domain that can be evaluated by the classical BDWF approach or by numerical methods. The above response was then used to derive the seismic motion of a capped pile group by making use of equilibrium and compatibility conditions at the piles' heads. Although it neglects the dynamic interaction between piles for vertical loads, the proposed solution yields very accurate results, as confirmed by the favourable comparison with rigorous analyses.

For a pair of piles the amplitude of the rotational motion increases with decreasing pile axial stiffness and pile spacing. For a row of piles, the rocking response is controlled by the distance between the first and last pile in the row.

A new transfer function from soil surface acceleration in free-field to mass acceleration of a SDOF system supported on a capped pile group was introduced, by taking into account the rocking motion generated by piles.

Finally, the approach was applied to a case study of a multi-storey building supported on cast-*in-situ* piles embedded in soft soil. The reduction of the horizontal component of the FIM was found to be very significant, in agreement with the acceleration recordings available at foundation level. The oscillation period of the structure is such that the reduction of spectral acceleration due to the filtering effect is negligible. On the other hand, the rocking motion induced by piles yields to an increase of the spectral acceleration of 7-10%, due to the occurrence of resonance between the multi-storey building and the vibrational rocking motion of the foundation. However, this is the case of a relatively tall building supported by only two alignments of piles, where the rocking motion was expected to be relevant. For large piled rafts, this effect is usually irrelevant, because the magnitude of the rotation at foundation level induced by kinematic interaction decreases quadratically with the raft width. Care must be taken in the case of slender structures with deep-seated foundations, such as bridge piers, silos, chimneys, and wind turbines, where the rocking component of the seismic input may be crucial for a reliable prediction of the related seismic response. In all these cases, the proposed approach may be helpful to quantify the effect of the rocking motion at foundation level induced by the piles.

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APPENDIX A – Pair of piles

A vertical pile loaded at its top by a horizontal force and a moment experiences a horizontal displacement and a rotation. The relationship among these quantities may be expressed in the form:

$$\begin{bmatrix} H \\ M \end{bmatrix} = \begin{bmatrix} K_{\mathrm{HH}} & K_{\mathrm{MH}} \\ K_{\mathrm{HM}} & K_{\mathrm{MM}} \end{bmatrix} \begin{bmatrix} u_{\mathrm{M}} \\ \theta_{\mathrm{M}} \end{bmatrix} \tag{A1}$$

where K_{HH}, $K_{\mathrm{MH}} = K_{\mathrm{HM}}$ and K_{MM} are the horizontal, coupled horizontal-rotational and rotational stiffness of the single pile.

For the case in Figure 3, as $H = 0$ due to the absence of a horizontal force loading the pile cap, u_{M} and θ_{M} can be expressed as:

$$u_{\mathrm{M}} = -\frac{K_{\mathrm{HM}}}{K_{\mathrm{HH}}}\theta_{\mathrm{M}} \tag{A2}$$

$$\theta_{\mathrm{M}} = \frac{M}{-\dfrac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \tag{A3}$$

The horizontal displacement and the rotation of the cap obtained from the sum of substructures (b) and (c) are:

$$u = u_{\mathrm{s}} + u_{\mathrm{M}} = u_{\mathrm{s}} - \frac{K_{\mathrm{HM}}}{K_{\mathrm{HH}}}\theta_{\mathrm{M}} \tag{A4}$$

$$\theta = \theta_{\mathrm{s}} + \theta_{\mathrm{M}} = \theta_{\mathrm{s}} + \frac{M}{-\dfrac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \tag{A5}$$

To satisfy equilibrium, the sum of the moments on the pile heads, $2M$, must balance the moment due to the axial forces, N:

$$2M = N \cdot s \tag{A6}$$

where s is the pile spacing.

If pile-to-pile interaction effects are not taken into account, the axial force N on the piles can be related to pile displacement w through single pile axial stiffness K_{V}:

$$N = K_{\mathrm{V}} \cdot w \tag{A7}$$

As:

$$w = -\theta \frac{s}{2} \tag{A8}$$

the rotation of the cap may be obtained substituting Eqs. (A6), (A7), and (A8) into Eq. (A5):

$$\theta = \frac{\theta_{\mathrm{s}}}{1 + \frac{1}{4} \frac{K_{\mathrm{V}} s^2}{-\frac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}}} \tag{A9}$$

Introducing a dimensionless factor ξ:

$$\xi = \frac{1}{4} \frac{K_{\mathrm{V}} s^2}{-\frac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \tag{A10}$$

the above kinematical quantities may be conveniently summarised as:

$$\theta_{\mathrm{M}} = -\frac{\xi}{1+\xi} \theta_{\mathrm{s}} \tag{A11}$$

$$\theta = \frac{\theta_{\mathrm{s}}}{1+\xi} \tag{A12}$$

$$u = u_{\mathrm{s}} + \frac{\xi}{1+\xi} \frac{K_{\mathrm{HM}}}{K_{\mathrm{HH}}} \theta_s \tag{A13}$$

APPENDIX B - Group of equally spaced piles

In this case, moment equilibrium writes:

$$nM = \sum_{i=1}^{n} N_i x_i \tag{B1}$$

As downwards displacements are positive (see Figure 6), N_i and w_i have always the same sign:

$$N_i = K_{\mathrm{V}i} \cdot w_i \tag{B2}$$

The vertical displacement w_i varies linearly with abscissa x_i:

$$w_i = \theta\left(\frac{B}{2} - x_i\right) \tag{B3}$$

Substituting Eqs. (B2) and (B3) into Eq. (B1):

$$nM = \sum_{i=1}^{n} K_{\mathrm{V}i}\theta\left(\frac{B}{2} - x_i\right)x_i = \theta\sum_{i=1}^{n} K_{\mathrm{V}i}\left(\frac{B}{2}x_i - x_i^2\right) \tag{B4}$$

If all piles have the same axial stiffness, moment equilibrium can be written as:

$$nM = \theta K_{\mathrm{V}}\sum_{i=1}^{n}\left(\frac{B}{2}x_i - x_i^2\right) = \theta K_{\mathrm{V}} s^2 \sum_{i=1}^{n}\left[\frac{n-1}{2}(i-1) - (i-1)^2\right] \tag{B5}$$

which leads to:

$$\theta_{\mathrm{M}} = -\frac{\xi f(n)}{1+\xi f(n)}\theta_{\mathrm{s}} \tag{B6}$$

$$\theta = \frac{\theta_s}{1+\xi f(n)} \tag{B7}$$

$$u = u_{\mathrm{s}} + \frac{\xi f(n)}{1+\xi f(n)}\frac{K_{\mathrm{HM}}}{K_{\mathrm{HH}}}\theta_{\mathrm{s}} \tag{B8}$$

where:

$$f(n) = \frac{n^2-1}{3} \tag{B9}$$

APPENDIX C - Groups of unevenly distributed piles

The horizontal displacements along the *X*-axis and the rotation about the *Y*-axis can be written as:

$$u_{\mathrm{X}} = u_{\mathrm{S}} + u_{\mathrm{MX}} = u_{\mathrm{S}} - \frac{K_{\mathrm{HM}}}{K_{\mathrm{HH}}}M_{\mathrm{Y}} \tag{C1}$$

$$\theta_{\mathrm{Y}} = \theta_{\mathrm{s}} + \theta_{\mathrm{MY}} = \theta_{\mathrm{s}} + \frac{M_{\mathrm{Y}}}{-\frac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \tag{C2}$$

Moment equilibrium yields:

$$pM_{\mathrm{Y}} = \sum_{i=1}^{p} N_i x_i \tag{C3}$$

where p is the total number of piles and x_i the abscissa of i-th pile in the reference system of Figure 8. Piles are supposed to be identical, and hence:

$$N_i = K_V \cdot u_{Zi} \tag{C4}$$

where u_{Zi} is the vertical displacement of i-th pile. This last quantity can be written as:

$$u_{Zi} = -\theta_Y \left(x_i - x_G \right) \tag{C5}$$

where:

$$x_G = \sum_{i=1}^{p} \frac{x_i}{p} \tag{C6}$$

The overall rotation of the capped pile group, θ_Y, can be obtained by substituting Eqs. (C4), (C5) and (C6) into Eq. (C2):

$$\theta_Y = \frac{\theta_s}{1 + \dfrac{K_V}{-\dfrac{K_{HM}^2}{K_{HH}} + K_{MM}} \displaystyle\sum_{i=1}^{p} \frac{x_i^2 - x_i x_G}{p}} \tag{C7}$$

As for the pair of piles, a dimensionless parameter can be introduced:

$$\xi_Y = \frac{K_V}{-\dfrac{K_{HM}^2}{K_{HH}} + K_{MM}} \sum_{i=1}^{p} \frac{x_i^2 - x_i x_G}{p} \tag{C8}$$

The overall rotation and displacement of the capped pile group can be finally derived:

$$u_X = u_s + \frac{\xi_Y}{1+\xi_Y} \frac{K_{HM}}{K_{HH}} \theta_s \tag{C9}$$

$$\theta_Y = \frac{\theta_s}{1+\xi_Y} \tag{C10}$$

For a seismic action parallel to the Y-axis, the kinematical quantities of the capped pile group write:

$$u_Y = u_s + \frac{\xi_X}{1+\xi_X} \frac{K_{HM}}{K_{HH}} \theta_s \tag{C11}$$

$$\theta_X = \frac{\theta_s}{1+\xi_X} \tag{C12}$$

where

$$\xi_{\mathrm{X}} = \frac{K_{\mathrm{V}}}{-\frac{K_{\mathrm{HM}}^2}{K_{\mathrm{HH}}} + K_{\mathrm{MM}}} \sum_{i=1}^{p} \frac{y_i^2 - y_i y_{\mathrm{G}}}{p} \tag{C13}$$

$$y_{\mathrm{G}} = \sum_{i=1}^{p} \frac{y_i}{p}. \tag{C14}$$

APPENDIX D

Suppose that a Single Degree Of Freedom (SDOF) system, characterized by mass m_{S}, stiffness k_{S} and viscous damping coefficient c_{S}, is excited by a base acceleration a_{b} with excitation frequency ω under free-field conditions. The absolute acceleration of the structural mass, a_{st}, can be expressed as:

$$a_{\mathrm{st}} = \left[1 + \omega^2 H(\omega) \cdot m_{\mathrm{S}}\right] \cdot a_{\mathrm{b}} = F(\omega) \cdot a_{\mathrm{b}} \tag{D1}$$

where

$$H(\omega) = \frac{1}{-m_{\mathrm{S}}\omega^2 + i c_{\mathrm{S}}\omega + k_{\mathrm{S}}}. \tag{D2}$$

If the rotational component of the base motion induced by kinematic interaction is taken into account, the structural displacement of the mass, u_{st}, relative to its base, is the algebraic sum of the displacement induced by the translational component, u_1, and that generated by the rotational vibration, u_2.

The translational component must satisfy the equation of motion:

$$m_{\mathrm{S}}\ddot{u}_1 + c_{\mathrm{S}}\dot{u}_1 + k_{\mathrm{S}}u_1 = -m_{\mathrm{S}}a_b \tag{D3}$$

where a_{b} has been already defined. The solution of this equation is well-known and can be conveniently put in the form:

$$u_1(\omega) = -m_{\mathrm{S}}H(\omega)a_{\mathrm{b}} = m_{\mathrm{S}}\omega^2 H(\omega)u_{\mathrm{b}} \tag{D4}$$

where u_{b} is the absolute displacement of the base. The rotational component u_2 can be further decomposed:

$$u_2 = -\theta_b h + u_{2R} \tag{D5}$$

where u_{2R} is the elastic component of the structural displacement generated by the rotational oscillation, θ_b the base rotation and h the height of the mass. This latter component must satisfy the following equation of dynamic equilibrium:

$$m_S \ddot{u}_{2R} + c_S \dot{u}_{2R} + k_S u_{2R} = m_S h \ddot{\theta}_b \tag{D6}$$

whose solution is:

$$u_{2R} = m_S h H(\omega) \ddot{\theta}_b = -m_S h \omega^2 H(\omega) \theta_b \tag{D7}$$

It follows that the total displacement generated by the application of both a rotational and a translational harmonic excitation can be expressed as:

$$u_{st} = u_1 + u_2 - \theta_b h = -m_S \omega^2 H(\omega) u_b - \theta_b h \left(1 + m_S \omega^2 H(\omega)\right) = \left(F(\omega) - 1\right) u_b - F(\omega) \theta_b h \tag{D8}$$

The ratio of the absolute structural acceleration generated by the two components of the FIM, a_{SK}, over that generated under free-field, a_{sff}, can be finally evaluated:

$$J_a = \frac{-\omega^2 u_{st} + a_b}{F(\omega) a_{ff}} = \frac{-\omega^2 \left(F(\omega) - 1\right) u_b + \omega^2 \theta_b h F(\omega) + a_b}{F(\omega) a_{ff}} = \frac{a_b F(\omega)}{F(\omega) a_{ff}} + \frac{\omega^2 \theta_b h F(\omega)}{F(\omega) a_{ff}} = I_u - \frac{h}{d} I_\theta \tag{D9}$$

Table I. Main characteristics of the selected earthquakes

Event (year)	Station	Identifier	Mw	Epicentral Distance (km)	PGA (*g*)
Irpinia (1980)	Sturno	ASTU270	6.9	32	0.320
Kozani (1995)	Kozani Prefecture	KOZ342	6.5	17	0.208
Umbria-Marche (1997)	Nocera Umbra	ENCB090	5.5	10	0.383

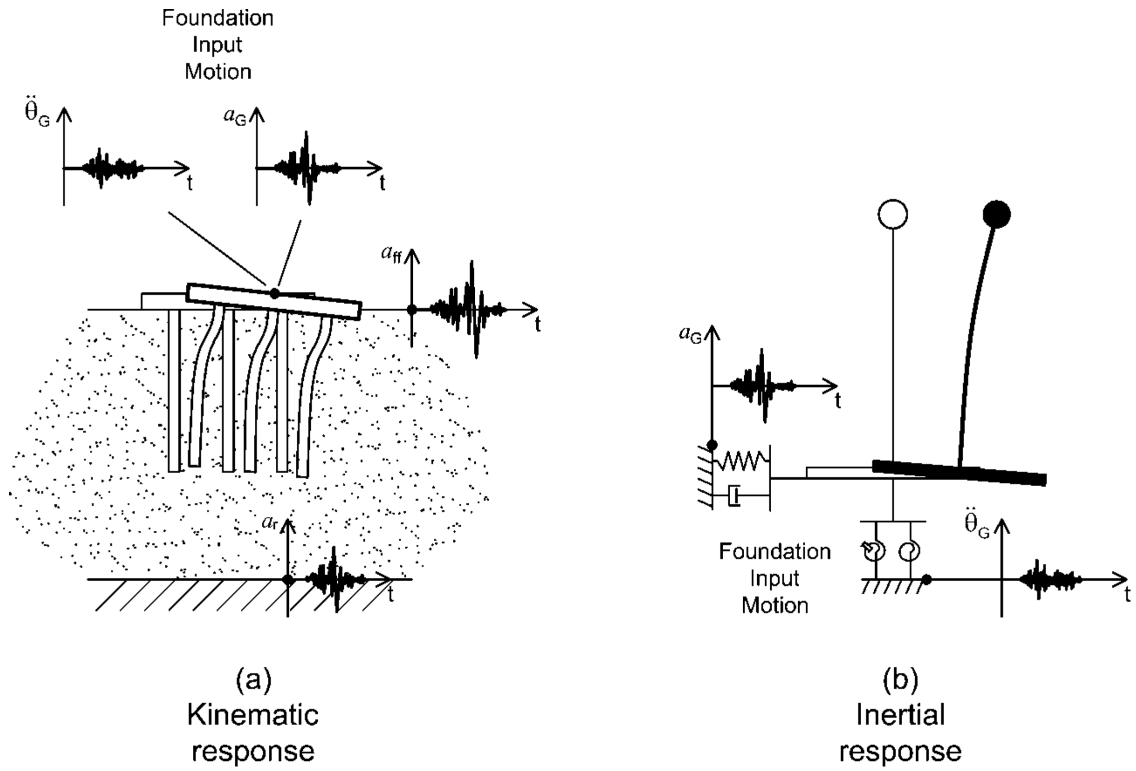

Figure 1. Problem definition: (a) kinematic response, (b) inertial response.

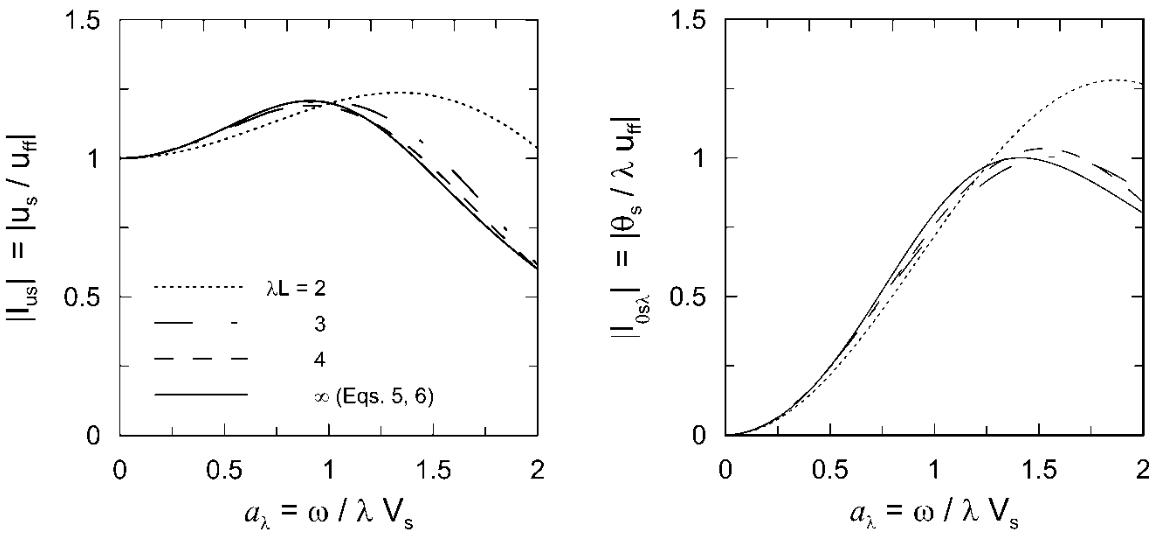

Figure 2. Effect of λL on the interaction factors.

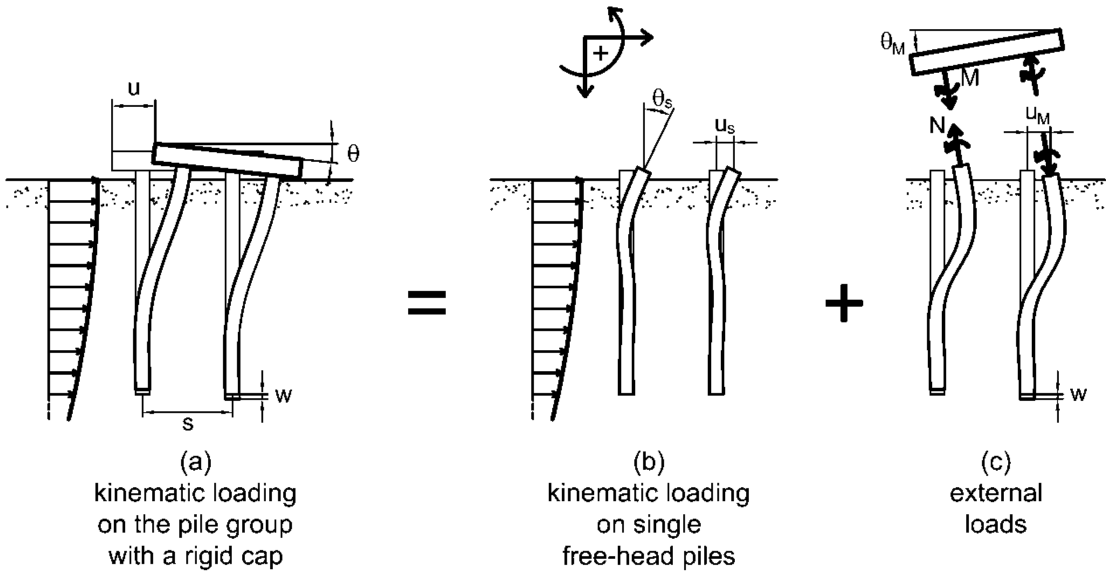

Figure 3. Schematic decomposition of the kinematic response of a pair of piles.

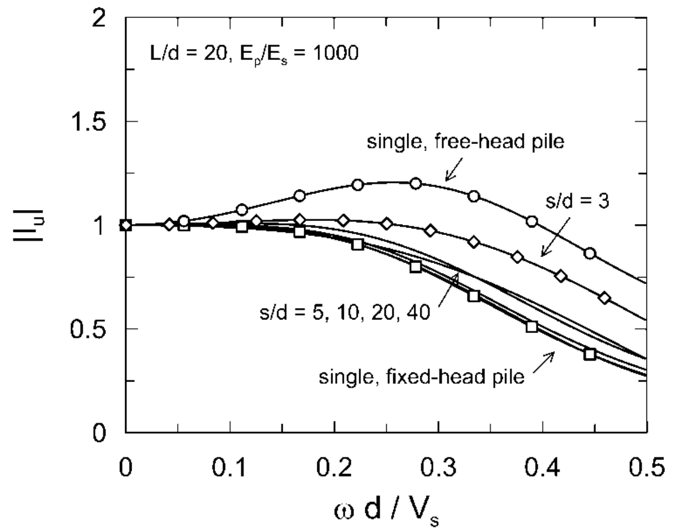

Figure 4. Kinematic interaction factor I_u against dimensionless frequency parameter a_0 as function of *s/d*.

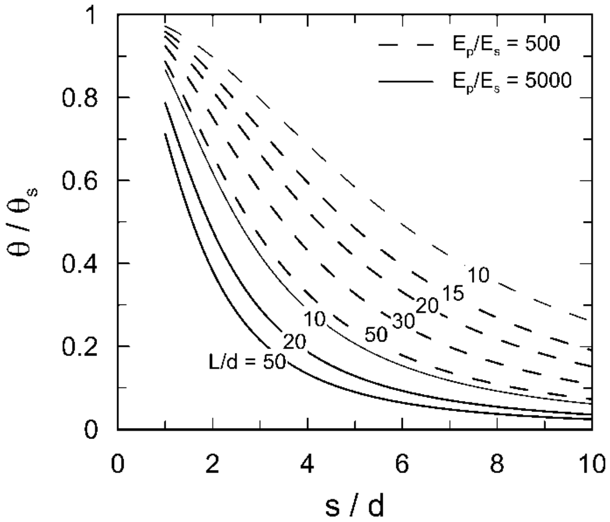

Figure 5. Normalized cap rotation θ/θ_s as function of *s/d* and *L/d*.

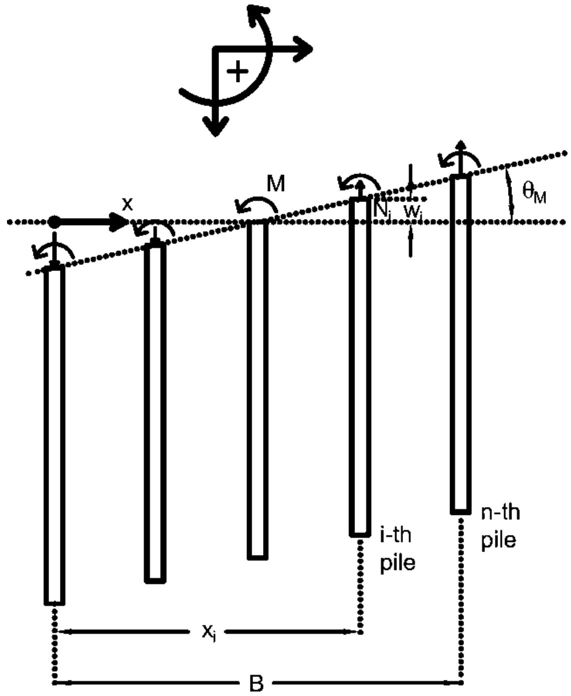

Figure 6. Row of equally spaced piles.

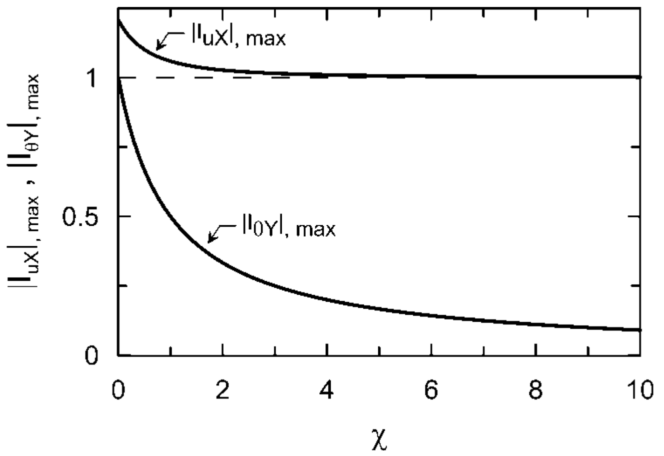

Figure 7. Interaction factors $I_{uX,max}$ and $I_{\theta Y,max}$ as function of χ.

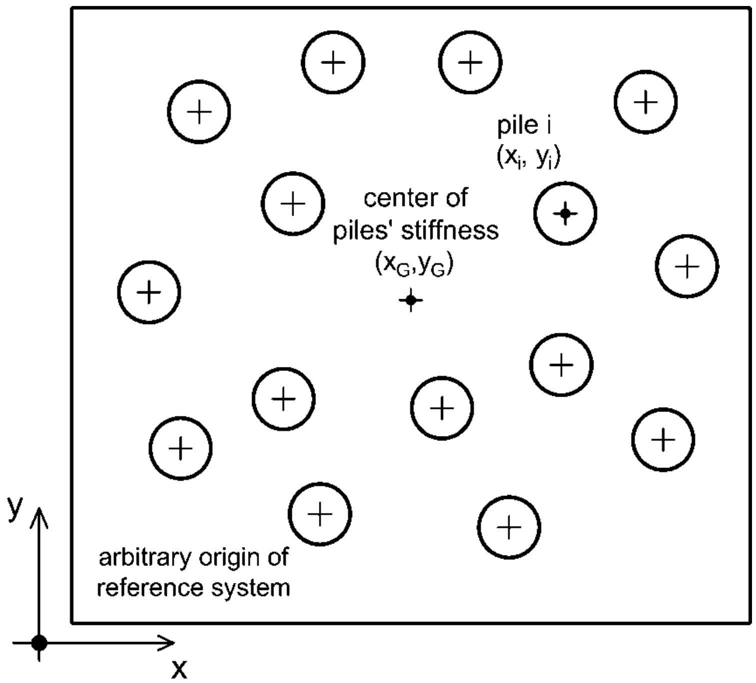

Figure 8. Group of unevenly distributed piles.

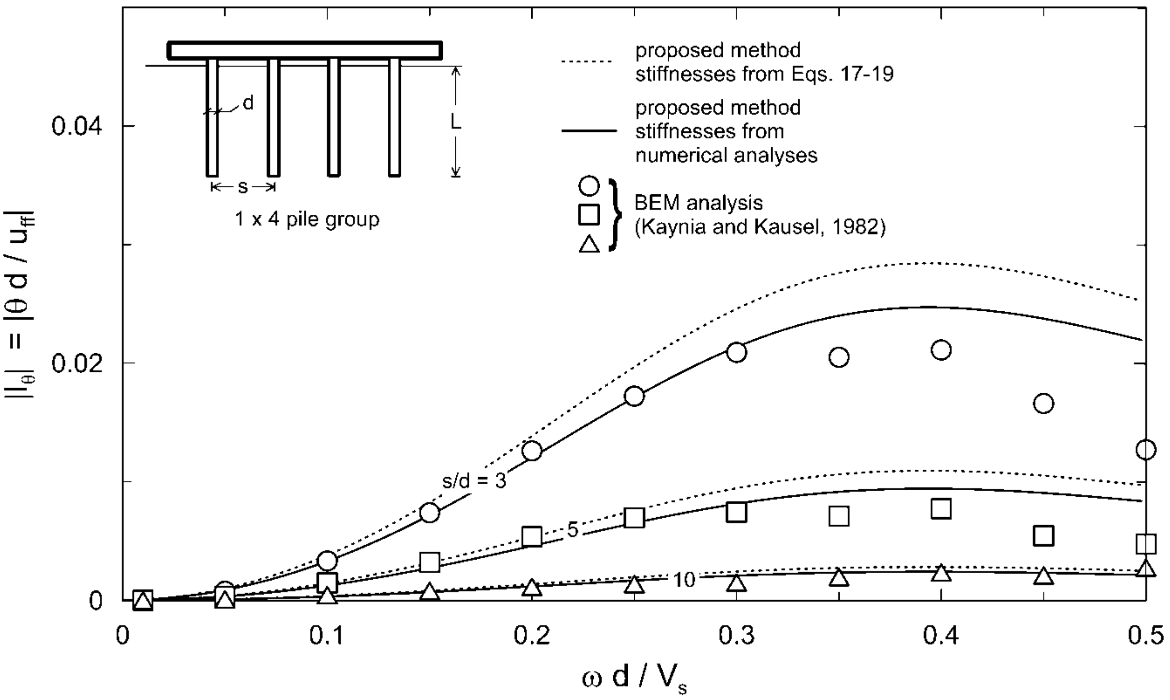

Figure 9. Comparison of the proposed analytical procedure against results from the 3D BEM code by Kaynia & Kausel (1982).

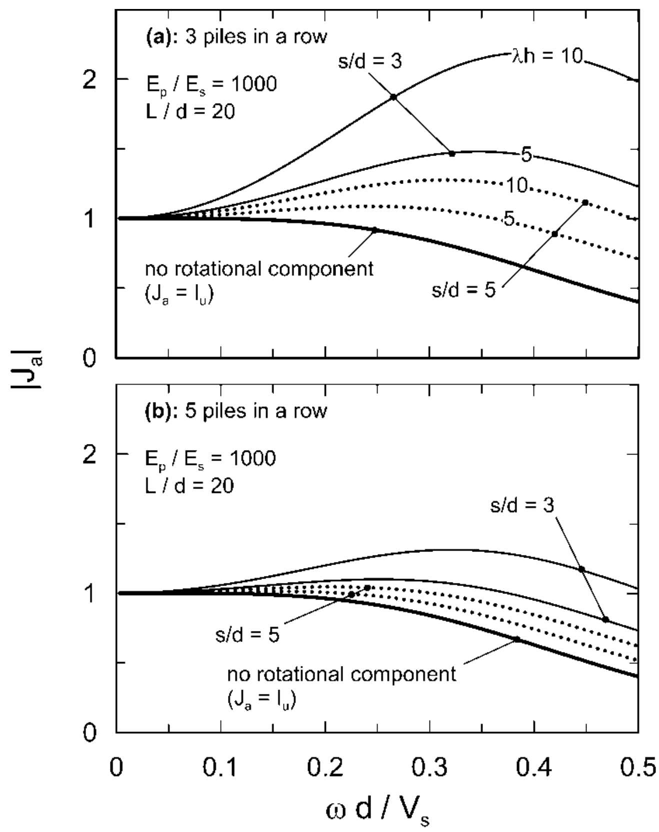

Figure 10. Effect of *s*/*d* and λh on the response of a SDOF system for (1×3) and (1×5) groups of piles.

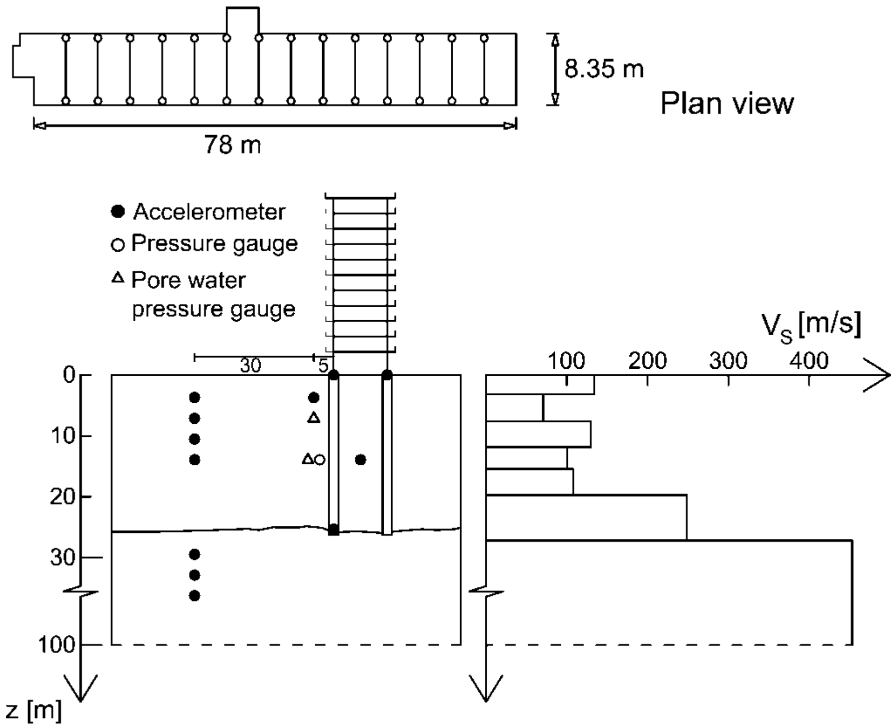

Figure 11. Case study: a 11-storey residential building in Japan (modified from Gazetas, 1984).

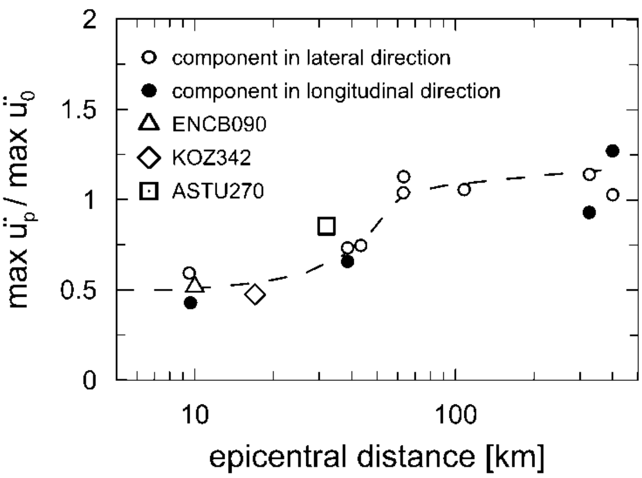

Figure 12. Ratio of peak acceleration at the top of the piles over the one at free-field as function of epicentral distance.

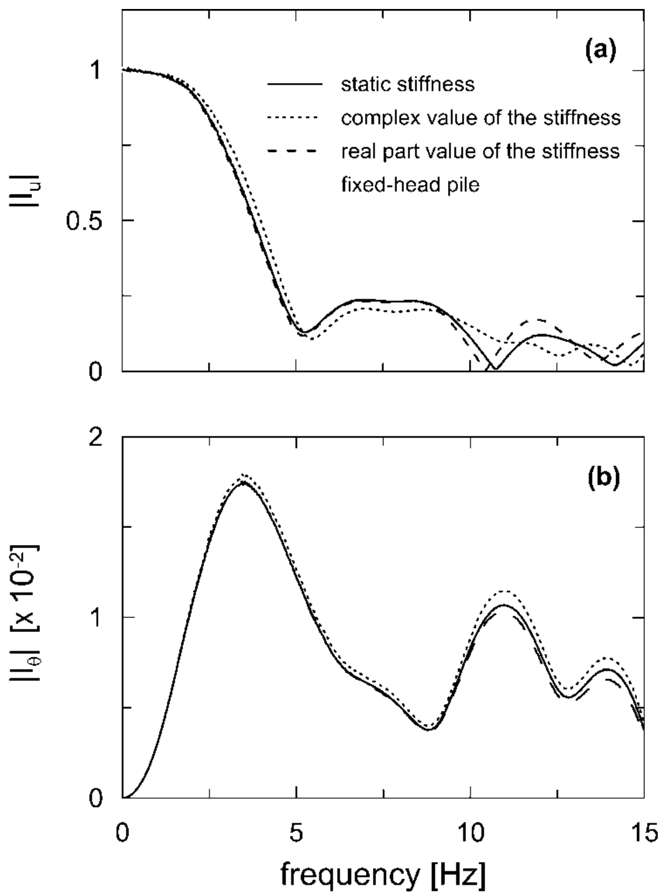

Figura 13. I_u and I_θ from frequency domain analysis.

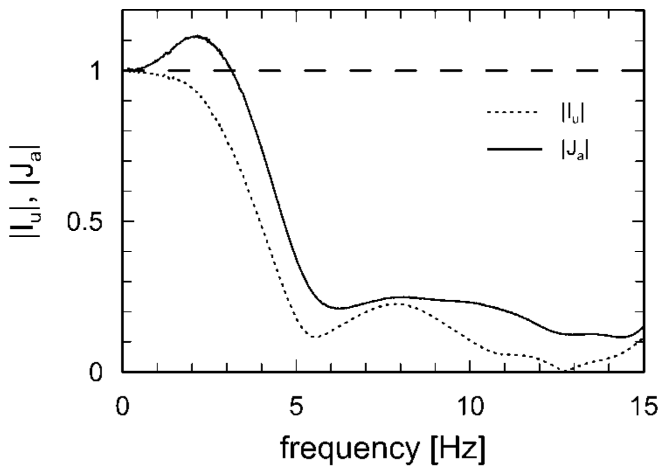

Figure 14. Comparison between J_a and I_u

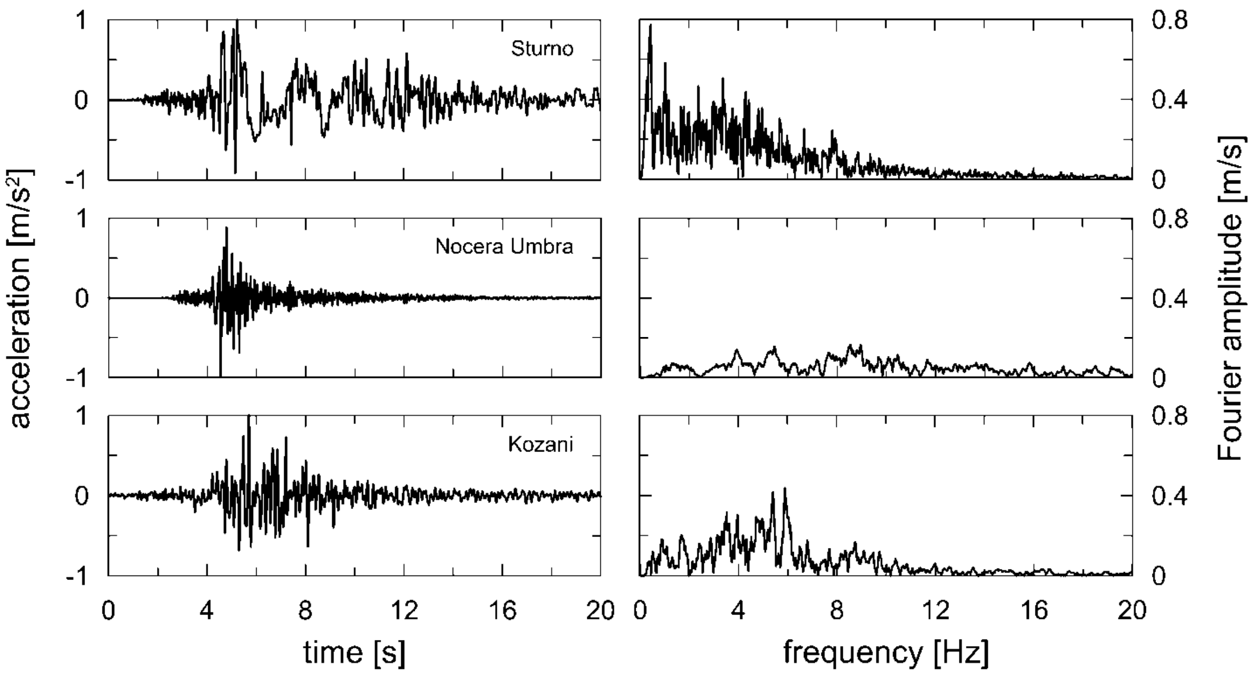

Figure 15. Acceleration time histories and Fourier Amplitude spectra of the selected earthquakes.

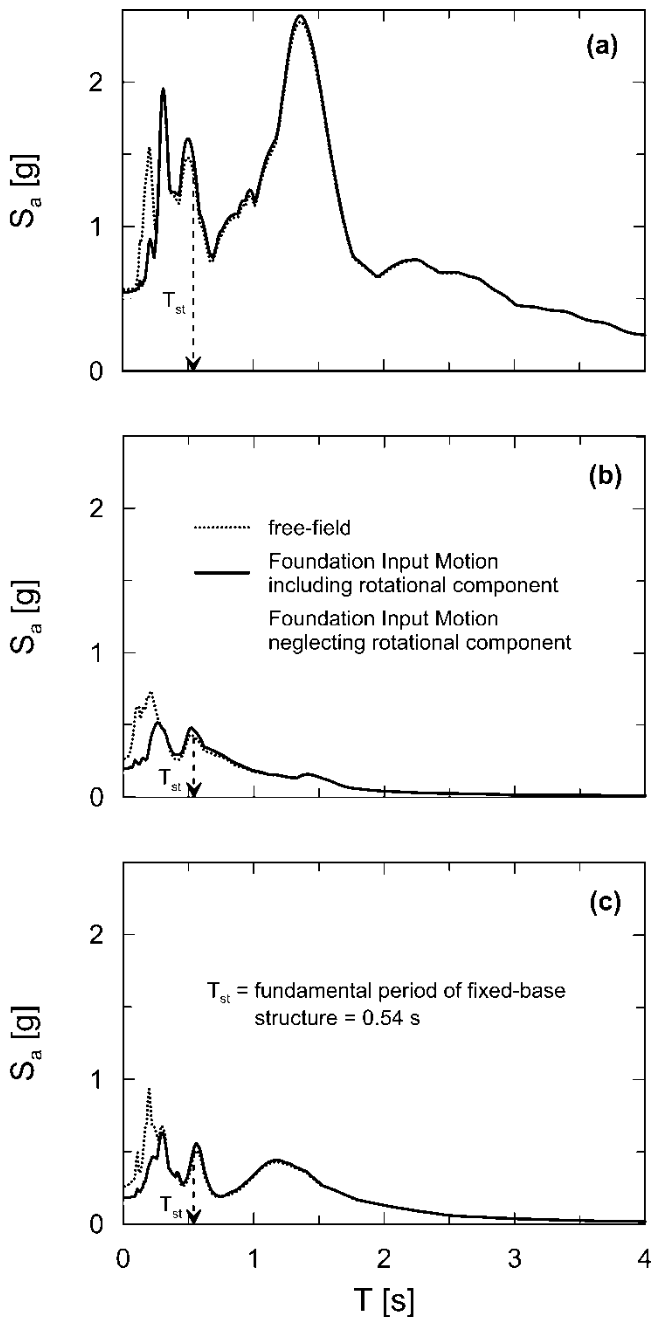

Figure 16. Response spectra for the three input signals: (a) Sturno; (b) Nocera Umbra; (c) Kozani.